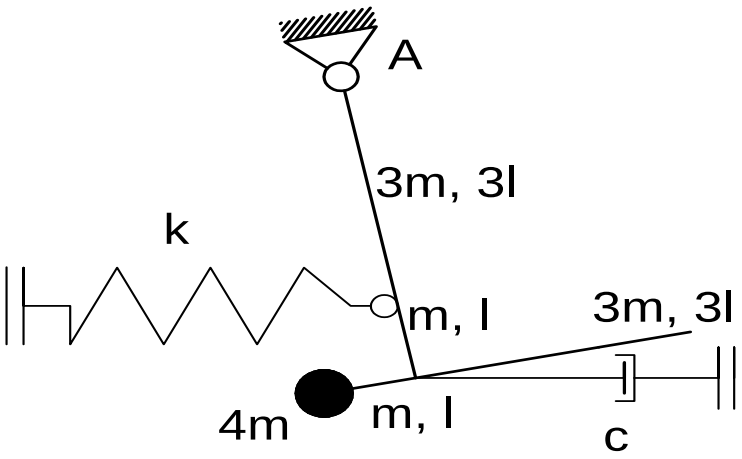
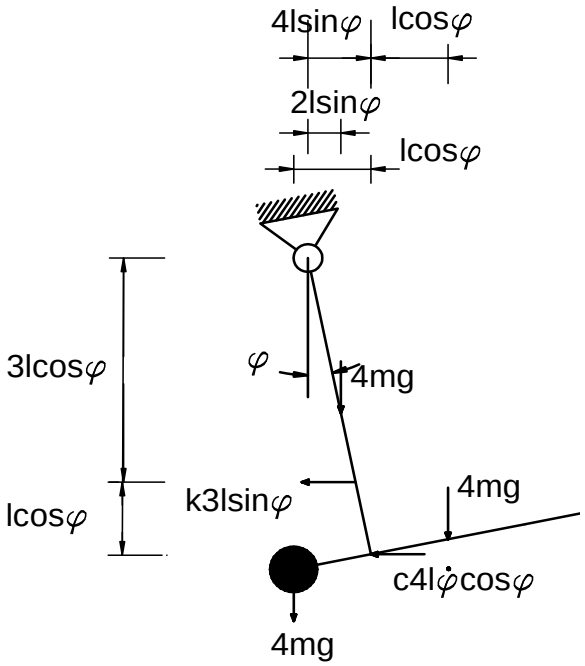
Zadatak 3.



$$\begin{aligned} m &= 10 \\ k &= 4\frac{kN}{m} \\ l &= 20m \\ c &= 200\frac{kg}{s} \end{aligned}$$

*** Diferencijalna jednačina kretanja:**

$$\begin{aligned} I_A \cdot \ddot{\varphi} &= \sum M_A \\ I_A &= \frac{1}{3}4m \cdot (4l)^2 + \frac{1}{12}4m \cdot (4l)^2 + 4m \cdot [l^2 + (4l)^2] + 4m \cdot [l^2 + (4l)^2] \\ &= \frac{488}{3}ml^2 = 162.67ml^2 \end{aligned}$$



$$\begin{aligned} \sum M_A &= -4mg \cdot 2l \cdot \sin\varphi \\ &- 4mg \cdot (4l\sin\varphi + l\cos\varphi) \\ &- 4mg \cdot (4l\sin\varphi - l\cos\varphi) \\ &- k3l\sin\varphi 3l\cos\varphi \\ &- c4l\dot{\varphi}\cos\varphi 4l\cos\varphi \\ \sum M_A &= -40mgl\sin\varphi \\ &- 9kl^2\sin\varphi\cos\varphi \\ &- 16cl^2\dot{\varphi}\cos^2\varphi \end{aligned}$$

*** Diferencijalna jednačina kretanja:**

$$\begin{aligned}\frac{488}{3}ml^2\ddot{\varphi} &= -40mgl\sin\varphi - 9kl^2\sin\varphi\cos\varphi - 16cl^2\dot{\varphi}\cos^2\varphi \\ \ddot{\varphi} &= -\frac{15}{61}\frac{g}{l}\sin\varphi - \frac{27}{488}\frac{k}{m}\sin\varphi\cos\varphi - \frac{6}{61}\frac{c}{m}\dot{\varphi}\cos^2\varphi\end{aligned}$$

*** Male oscilacije:** $cos\varphi \approx 1, \sin\varphi \approx \varphi$

$$\begin{aligned}\ddot{\varphi} &= -\frac{15}{61}\frac{g}{l}\varphi - \frac{27}{488}\frac{k}{m}\varphi - \frac{6}{61}\frac{c}{m}\dot{\varphi} \\ \ddot{\varphi} &+ \frac{6}{61}\frac{c}{m}\dot{\varphi} + \varphi(\frac{15}{61}\frac{g}{l} + \frac{27}{488}\frac{k}{m}) = 0\end{aligned}$$

*** Kružna frekvencija neprigušenih oscilacija:**

$$\omega = \sqrt{\frac{15}{61}\frac{g}{l} + \frac{27}{488}\frac{k}{m}} = \sqrt{\frac{15}{61}\frac{10}{20} + \frac{27}{488}\frac{4000}{10}} = 4.717\frac{rad}{s}$$

*** Period oscilovanja:**

$$T = \frac{2\pi}{\omega} = 1.33s$$

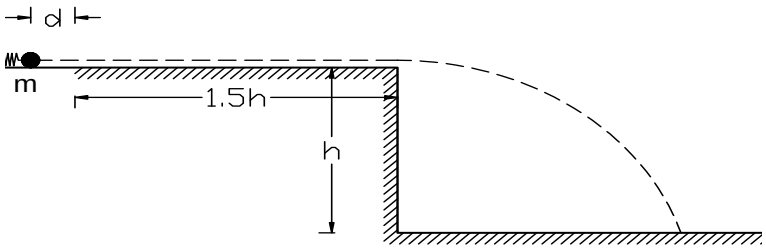
*** Relativno prigušenje:**

$$\begin{aligned}2\omega\xi &= \frac{6}{61}\frac{c}{m} \\ \xi &= \frac{3}{61}\frac{c}{m\omega} = \frac{3}{61}\frac{200}{10\cdot 4.717} = 0.209\end{aligned}$$

*** Period oscilovanja prigušenog sistema:**

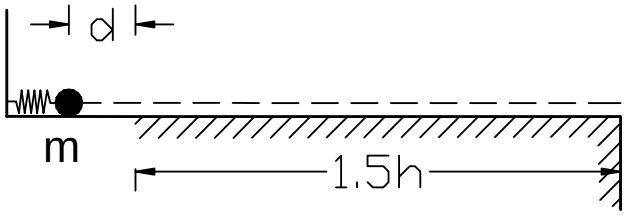
$$T_d = \frac{T}{\sqrt{1-\xi^2}} = \frac{1.33}{\sqrt{1-0.209^2}} = 1.36$$

Zadatak 4.



$$\begin{aligned} m &= 0.3kg \\ h &= 6m \\ \delta &= 0.08m \\ k &= 7\frac{kN}{m} \end{aligned}$$

a)



$$\begin{aligned} T_B - T_A &= A_{A-B} \\ 0 &= \frac{1}{2}k\delta^2 - \mu_{max}mg \cdot 1.5h \\ \mu_{max} &= 0.846 \end{aligned}$$

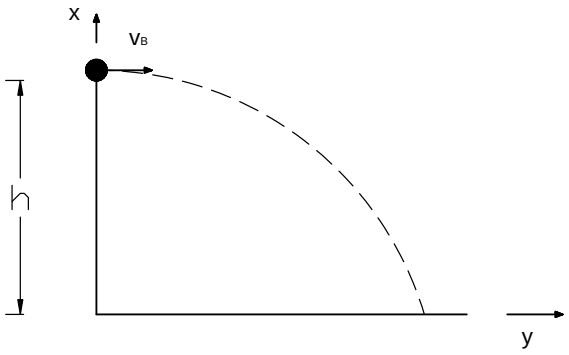
b)

$$\begin{aligned} \mu &= 0.3 \cdot 0.846 = 0.234 \\ T_B - T_A &= A_{A-B} \\ \frac{1}{2}m \cdot v_B^2 &= \frac{1}{2}k\delta^2 - \mu mg \cdot 1.5h \\ v_B &= 10.39\frac{m}{s} \end{aligned}$$

- **Slobodni pad**

$$\begin{aligned} m\ddot{x} = 0 &\Rightarrow \dot{x} = C_1 \Rightarrow x = C_1 \cdot t + C_2 \\ m\ddot{y} = -mg &\Rightarrow \dot{y} = -gt + C_3 \Rightarrow y = -\frac{gt^2}{2} + C_3 t + C_4 \end{aligned}$$

- **Početni uslovi:**



$$\begin{aligned} x(0) = 0 &\Rightarrow C_2 = 0 \\ y(0) = h &\Rightarrow C_4 = h \\ \dot{x}(0) = v_b &\Rightarrow C_1 = v_B \\ \dot{y}(0) = 0 &\Rightarrow C_3 = 0 \end{aligned}$$

- **Konačne jednačine slobodnog kretanja:**

$$x(t) = v_B \cdot t \tag{1}$$
$$y(t) = -\frac{gt^2}{2} + h \tag{2}$$

- **Trajektorija:**

$$(1) \Rightarrow t = \frac{x}{v_B} \tag{3}$$
$$(3) \rightarrow (2) \Rightarrow y = -\frac{1}{2}\frac{g}{v_B^2} \cdot x^2 + h$$

- **Pad tačke:**

  * **Vreme:**

$$\begin{aligned} y(t_P) &= 0 \\ -\frac{g}{2}{t_P}^2 + h &= 0 \Rightarrow t_P = 1.106s \end{aligned}$$

  * **Mesto:**

$$L = x(t_P) = 10.39 \cdot 1.106 = 11.49m$$

  * **Ugao pada:**

$$\begin{aligned} \dot{x} &= 10.39\frac{m}{s} \\ \dot{y} &= -gt_P = -9.81 \cdot 1.106 = -10.85\frac{m}{s} \\ \tan\alpha &= \frac{10.85}{10.39} \Rightarrow \alpha = 46.24 \end{aligned}$$